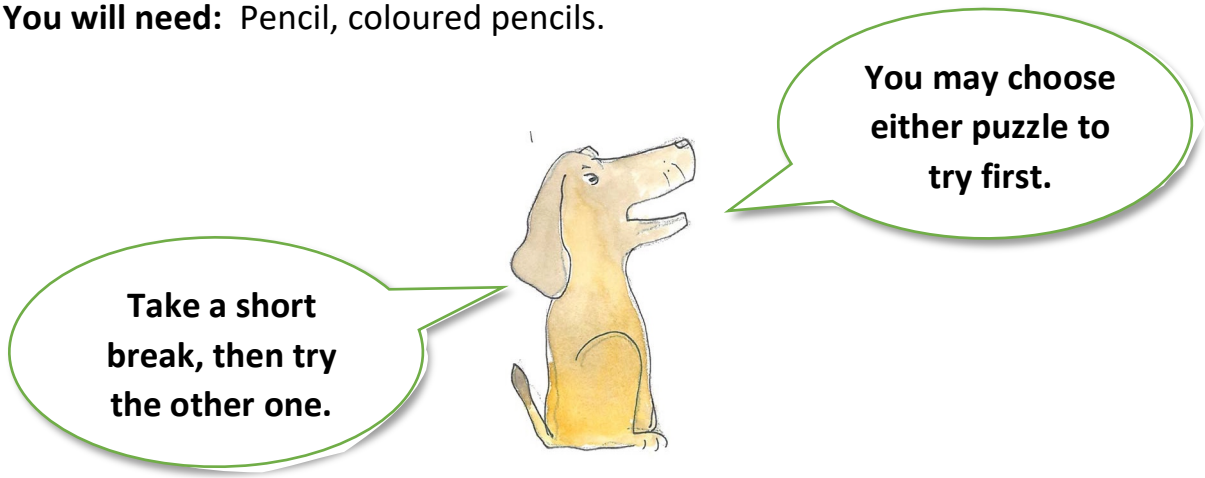
Mathematical investigation (1)

Investigating is a great way to learn to think mathematically, apply logic, spot patterns and improve our perseverance.

How many squares?

AIM: To investigate two geometric puzzles which involve drawing squares.

You will need: Pencil, coloured pencils.



A. How many squares on a dot grid?

Don't lift your pencil!

> *"Begin at the beginning," the King said gravely, "and go on until you come to the end; then stop."*
>
> Lewis Carroll, *Alice's Adventures in Wonderland.*

In this puzzle, you draw lines to join dots on a grid.
The aim is to find the largest number of squares possible.
BUT you may not lift your pencil… It must be a continuous line…

Rules

- You cannot lift your pencil and start from another place. It must be one long line.
- Draw straight horizontal (left or right) lines or vertical (up or down) lines.
- You cannot go over a line that has already been drawn.
- You can cross through the same dot in a different direction, e.g. first horizontally then vertically.

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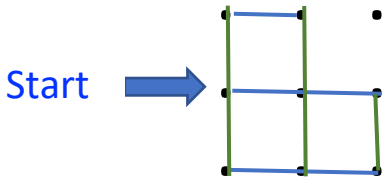
Example



In this example, the line starts **halfway** down on the **left.**
Draw the line **horizontally across 2** to the far right side, then **down 1.**
Next go **across 2** to the bottom left hand corner and then **up 2.**
Now **1 to the right and down 2.**
Now we are stuck! We have made 3 squares.
We think that it is not possible to draw a continuous line that will create 4 squares on a 3 by 3 dot grid…

Use the dot grids to try out your ideas. You can start on any dot – try starting at the centre or a corner…

Then try the **4 by 4** and **5 by 5 dot grids** (*see below for grids*). For each of those, what is the greatest number of small (1 by 1) squares possible?

Challenge

Try this using a **6 by 6** grid of dots!
Can you predict the maximum possible number of 1 by 1 squares?

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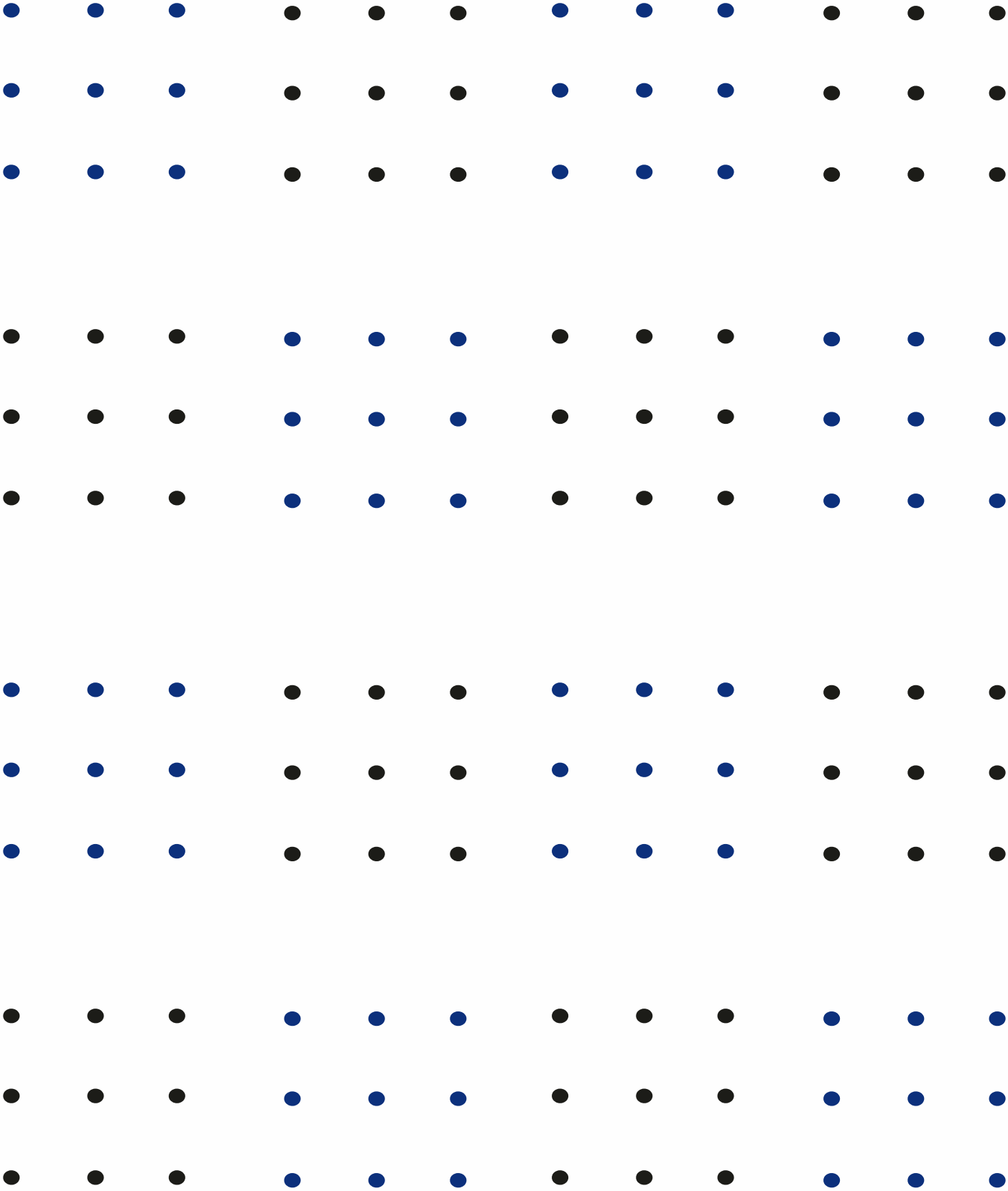

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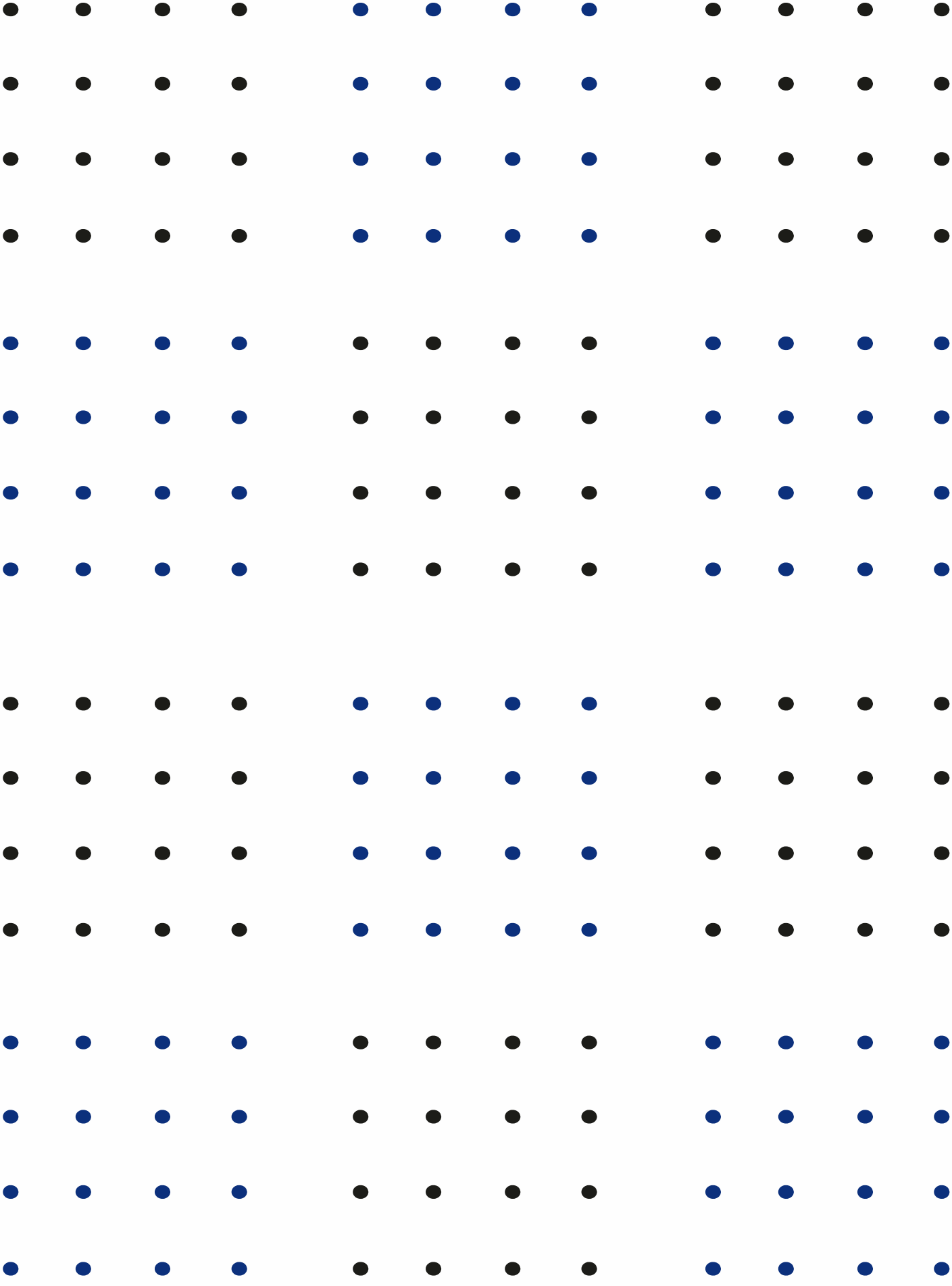

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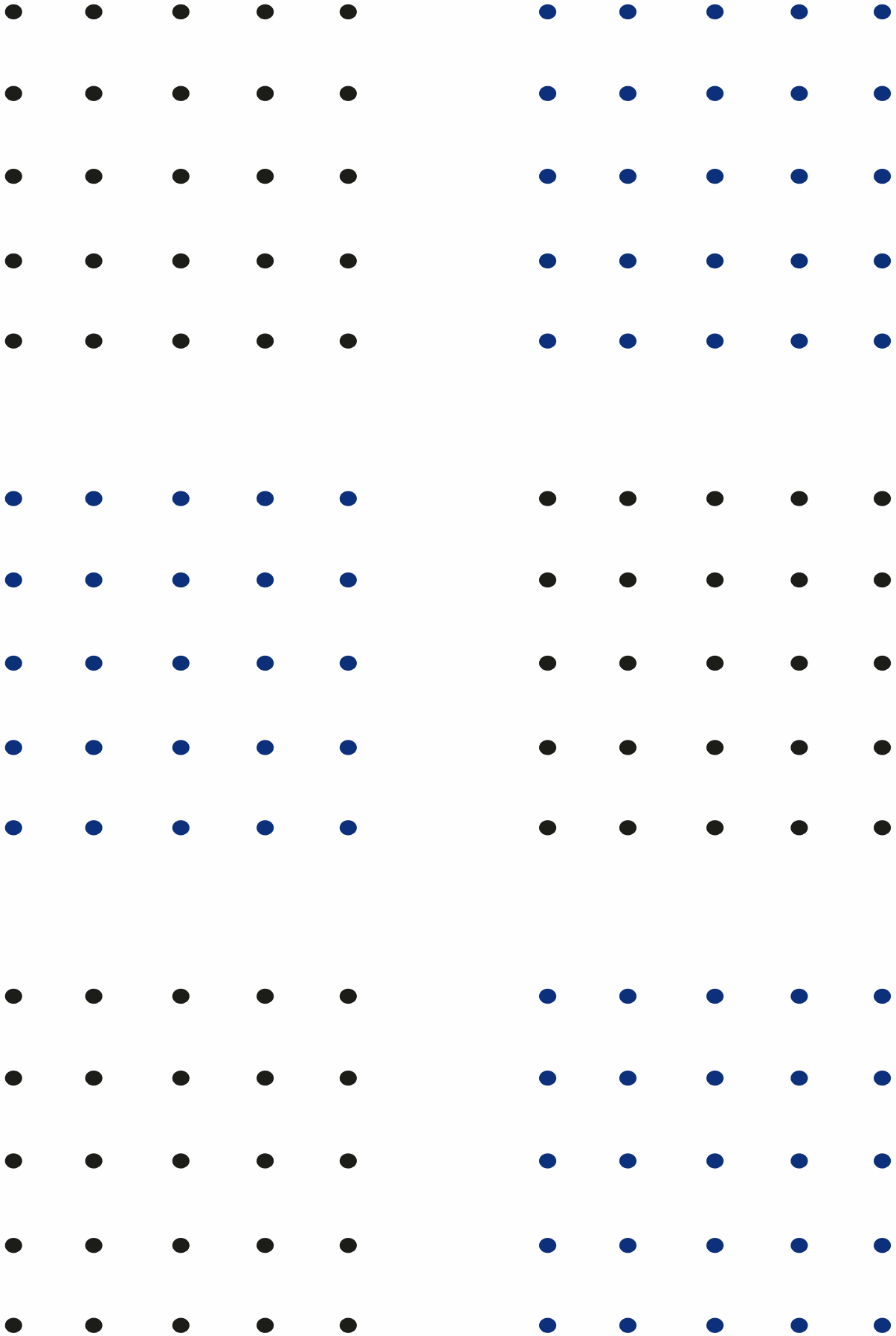

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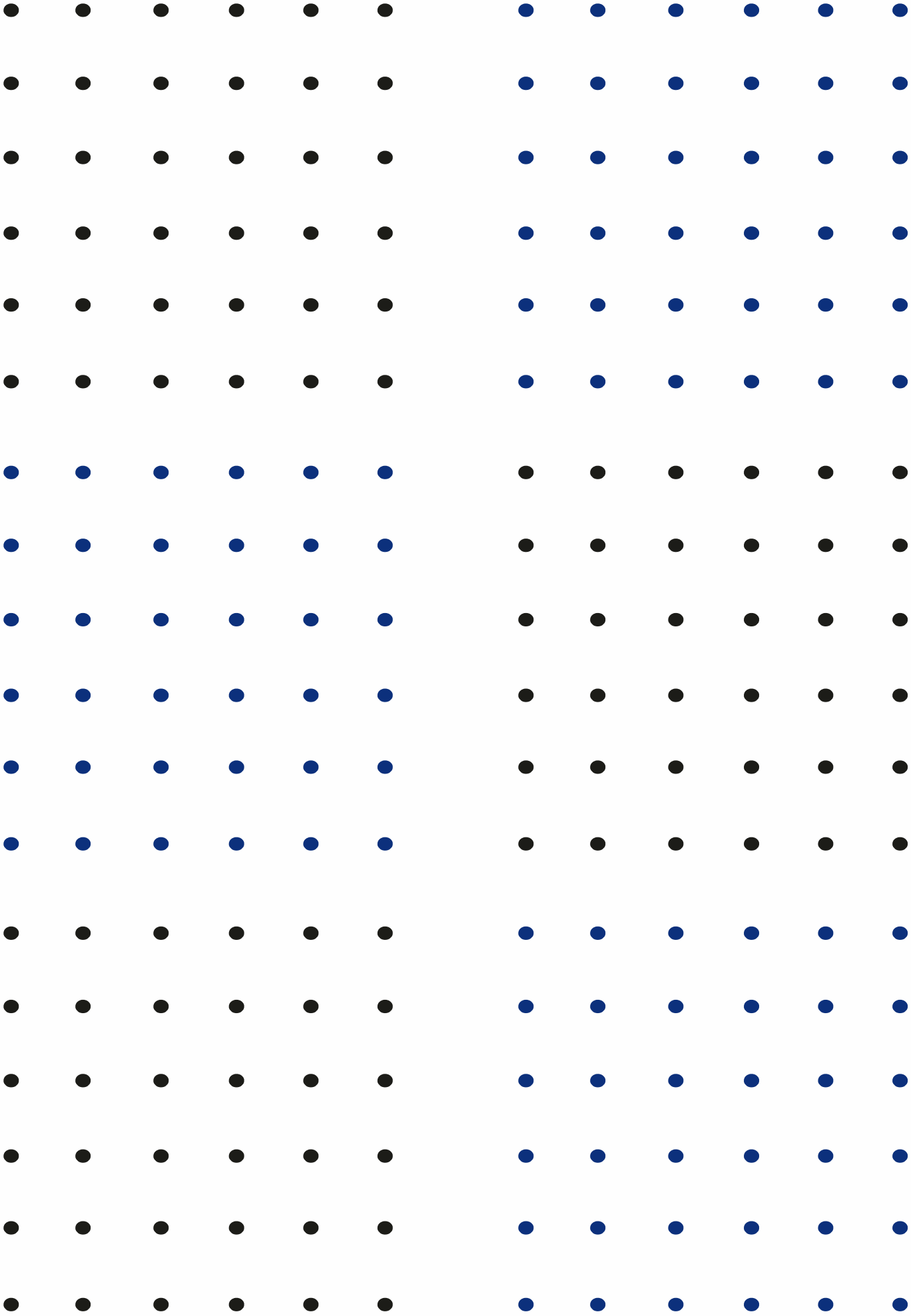

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B. How many squares in a square?

If you look at this 3 by 3 square of squares you will see that there are nine (1 by 1) squares...

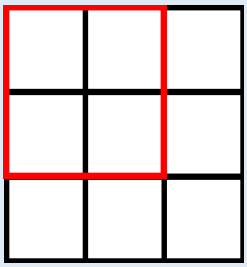
There are also four 2 by 2 squares, the size of the one in red.

So, altogether there are **14 squares in the square:**

one 3 by 3 square;

four 2 by 2 squares;

nine 1 by 1 squares.



Use the squared paper to investigate how many squares of each different size you can you can find in a 4 by 4 and 5 by 5 square. Make a note of any patterns.

Try laying out your findings as additions

3 x 3 square

1 + 4 + 9 = 14

4 x 4 square

1 + …

HINT

It might be useful to think about what type of number 1, 4, 9 and 16 are...

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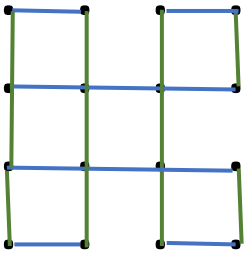
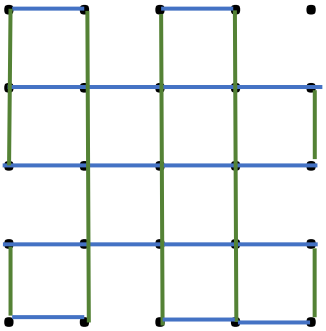
Solutions

Here is a solution for 4 by 4 grid that has 6 squares, and for 5 by 5 that has 11. In each case the start point is the 2nd dot down on the left. You may have found other solutions but these give the maximum number of small squares.





B. How many squares in a square?

A 4 by 4 square has **30 squares:** (1 + 4 + 9 + 16)

1, 4 by 4 square

4, 3 by 3 squares

9, 2 by 2 squares

16, 1 by 1 squares

A 4 by 4 square has **55 squares:** (1 + 4 + 9 + 16 + 25)

1, 5 by 5 square

4, 4 by 4 squares

9, 3 by 3 squares

16, 2 by 2 squares

25, 1 by 1 squares

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